# Personal View on Causation

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January 28, 2014

# Correlation does not always imply causation

- Number of storks and birth rate in Denmark.
- Ice cream consumption leads to murder.
- A pirate shortage causes global warming.
- Marriage increases men's life expectancy.
- etc.

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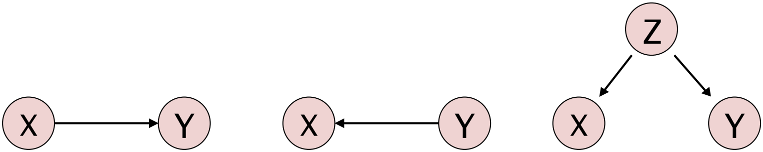
**Reisenbach Principle**
If there is a dependency between $X$ and $Y$, then




Any dependency is naturally **causal** ?

# Definition of Cause and Effect

**Asymmetry** (*structural equation model, functional causal model*)

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- "we may define a cause to be an object, followed by another, ... where if the first object had not been, the second had never existed (Hume, 1748, page 115)"
- "... an association may be classed as presumptively causal when it is believed that, *had the cause been* altered, the effect *would have been* changed (MacMahon and Pugh, 1967, page 12)."

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**"Counterfactual"**

*... had the cause been altered, the effect would have been changed ...*

# Experiment

$T \in \{0,1\}$: treatment (asteroid), $Y \in [0,1]$: outcome (survival)

$Y_0$ outcome the subject would have if received *control*
$Y_1$ outcome the subject would have if received *treatment*

| No. | Unit | $Y_1$ | $Y_0$ | $Y_1 - Y_0$ |
|---|---|---|---|---|
| 1 |  | 0.95 | 0.05 | 0.90 |
| 2 |  | 0.88 | 0.12 | 0.76 |
| 3 |  | 0.85 | 0.15 | 0.70 |
| 4 |  | 0.90 | 0.10 | 0.80 |
| | | **0.895** | **0.105** | **0.79** |

**Causal effect:** $\Delta = \mathbb{E}[Y_1] - \mathbb{E}[Y_0] = \mu_1 - \mu_0$

# Fundamental Problem of Causal Inference

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| | | **?** | **?** | **?** |

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# Counterfactual Model

Based on observed data $(Y_i, T_i), i = 1, \ldots, n$, we want to estimate

$$\Delta = \mu_1 - \mu_0 = \mathbb{E}[Y_1] - \mathbb{E}[Y_0].$$

Averages among those observed treatments and controls

$$\widetilde{Y}_1 = \frac{1}{n_1} \sum_{i:T_i=1} Y_i, \qquad \widetilde{Y}_0 = \frac{1}{n_0} \sum_{i:T_i=0} Y_i,$$

That is, $\widetilde{Y}_1$ estimates $\mathbb{E}[Y|T = 1]$ and $\widetilde{Y}_0$ estimates $\mathbb{E}[Y|T = 0]$. But

$$\widetilde{\Delta} = \mathbb{E}[Y|T = 1] - \mathbb{E}[Y|T = 0] \neq \Delta$$

Thus, $\widetilde{\Delta}$ is in general **not** an *unbiased* estimate of $\Delta$.

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**Assumption:** $(Y_0, Y_1) \perp\!\!\!\perp T$ $(\Rightarrow \mathbb{E}[Y|T] = \mathbb{E}[Y])$

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We can estimate $\Delta$ using appropriate adjustment.

- balancing the covariate distributions
- regression modeling (estimate $\mathbb{E}[Y|T, X]$ using regression)
- inverse weighting
- matching via propensity score ($(Y_0, Y_1) \perp\!\!\!\perp T \mid e(X)$)

**Goal:** using $Y$, $T$, and $X$ to simulate the randomized experiment.

# Discussions

- The primary issue in causality is the **bias** from the unobserved **confounders**.
  - **Causal discovery/inference** – reduce the bias by adjusting for unobserved confounders.
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| Causal inference | = | empirical inference | + | "**assumptions**" |
|---|---|---|---|---|
| $X \rightarrow Y$ or $X \leftarrow Y$ | | P(X,Y) | | ??? |