Computing Functions of Random Variables via Reproducing Kernel Hilbert Space Representations

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Overview and Motivation

Problem Statement

Let X and Y be two independent random variables. We are interested in the random variable Z given by

$$Z = g(X, Y)$$

for some function $g : \mathcal{X} \times \mathcal{Y} \to \mathcal{Z}$. What is the distribution of Z?

Motivation

- Arithmetic operation on random variables
- Probabilistic programming
- Causal inference
- Etc.

Previous Works

Sum of random variables $Z = X + Y$

Let *psi* : $\mathbb{R}^2 \to \mathbb{R}^2$ given by $\psi(x, y) = (x, x + y) := (x, z)$.

$$
\begin{aligned}
f_Z(z) &= \int_{\mathbb{R}} f_{XZ}(x, z)\, dx \\
&= \int_{\mathbb{R}} f_{XY}(\psi^{-1}(x, z))\, dx \\
&= \int_{\mathbb{R}} f_{XY}(x, z - x)\, dx \\
&= \int_{\mathbb{R}} f_X(x) f_Y(z - x)\, dx \\
&= f_X \star f_Y(z)
\end{aligned}
$$

Product of random variables $Z = X \cdot Y$

The Mellin transform instead of Fourier transform.

Kernel Map

Kernel Mean Map